ON QUATERNIONS AND THEIR GENERALIZATION AND THE HISTORY OF THE EIGHT-SQUARE THEOREM. ADDENDA.

By L. E. Dickson.

H. Scheffler* gave an erroneous 16-square formula

$$(\Sigma a_i^2)(\Sigma b_i^2) = \Sigma A_i^2,\ A_3 = a_8b_6 - a_{15}b_{13} + \cdots,\ A_{12} = -a_8b_{13} + a_{15}b_6 + \cdots,$$

whence ΣA_i^2 has the term $-4a_8a_{15}b_6b_{13}$. The sign $+$ before $a_{15}b_{16}$ in A_2 is probably a misprint as it causes many cases of failures of the formula.

S. A. Corey communicated privately a special 16-square formula. In his identity† take $c_1 = c_2 = 1$ and let t_{2n-1} be conjugate to t_{2n} and r_{2n-1} conjugate to r_{2n}. Then $2S_8S_8' = S_{16}$, where S_r denotes a sum of r squares. Evidently $2S_8 = S_8 + S_8 = S_{16}'$. As each of the numbers the sum of whose squares is S_8' take the hypotenuse of a right triangle, whence $S_8' = S_{16}''$. Thus $S_{18}'S_{16}'' = S_{16}$.

* Die Polydimensionalen Grössen und die Vollkommenen Primzahlen, Braunschweig, 1880, p. 94.

† Amer. Math. Monthly, 26, 1919, 72.